Partiel du 11 mars 2015

20 pts + 3 pts hors barème

Partie I) Cinématique

$$\vec{v}_{G_1} = v_0 \vec{e_x} \quad \text{et} \quad \vec{\Omega} = \Omega \vec{e_3}$$

1) $$\vec{v}_{I,S/R} = \vec{v}_{G_1 S/R} + \vec{\Omega} \wedge \vec{GI} = v_0 \vec{e_x} + \Omega \vec{e_3} \wedge (-R\vec{e_y})$$
$$= (v_0 + R\Omega)\vec{e_x}$$ 2

2) $$\vec{v}_{g,S/R} = \vec{v}_{I_1 \in S/R} - \vec{v}_{I_2 \in sol/R} = \vec{v}_{I_1 \in S/R} = \vec{v}_{I,S/R} = (v_0 + R\Omega)\vec{e_x}$$ 1

CRSG $\Rightarrow v_0 = -R\Omega$ 1

Partie II) Éléments cinétiques

I) $$M = \underbrace{\pi R^2}_{S} \sigma_1 - \underbrace{\pi \left(\frac{R}{2}\right)^2}_{S\ \text{vide}} \sigma_1 + \underbrace{\pi \left(\frac{R}{2}\right)^2}_{\text{nouveau mat}} \sigma_2$$
$$= \pi \sigma_1 R^2 + \pi(\sigma_2 - \sigma_1)\left(\frac{R}{2}\right)^2 \quad \text{et} \quad \sigma_2 = 5\sigma_1$$
$$= \pi\sigma_1 R^2 + 4\sigma_1 \pi \left(\frac{R}{2}\right)^2 = 2\pi\sigma_1 R^2$$ 3

II 1) Position de C_1' $\Rightarrow$ solide évidé !

O origine arbitraire $$\pi R^2 \sigma_1 \vec{OC_1} = \frac{1}{4}\pi R^2 \sigma_1 \vec{OC_2} + \frac{3}{4}\pi R^2 \sigma_1 \vec{OC_1'}$$
$$O \equiv C_1 \Rightarrow \vec{C_1C_1'} = -\frac{1}{3}\vec{C_1C_2}$$
$$\text{or} \quad C_1C_2 = \frac{R}{2} \Rightarrow C_1'C_1 = \frac{R}{6}$$ 2

2) $$2\pi R^2 \sigma_1 \vec{OG} = \frac{3}{4}\pi R^2 \sigma_1 \vec{OC_1'} + \frac{5}{4}\pi R^2 \sigma_1 \vec{OC_2}$$ σ_2
$$O \equiv C_1 \Rightarrow 2\vec{C_1G} = \frac{3}{4}\vec{C_1C_1'} + \frac{5}{4}\vec{C_1C_2}$$
$$= -\frac{1}{4}\vec{C_1C_2} + \frac{5}{4}\vec{C_1C_2}$$
donc $$\vec{C_1G} = \frac{\vec{C_1C_2}}{2} \Rightarrow C_1G = \frac{R}{4}.$$ 2

III 1) • $$I_{S_0/C_1 3} = \frac{1}{2} m_{S_0} R^2 = \frac{1}{2}(\pi R^2 \sigma_1) R^2 = \frac{1}{2}\pi\sigma_1 R^4$$ 1

• $$I_{S_0/G3} = I_{S_0/C_1 3} + m_{S_0}\vec{C_1G}^2 = \frac{1}{2}\pi\sigma_1 R^4 + (\pi R^2 \sigma_1)\left(\frac{R}{4}\right)^2 = \frac{9}{16}\pi\sigma_1 R^4$$ 1

• $$I_{S_1'/C_2 3} = \frac{1}{2} m_{S_1'}\left(\frac{R}{2}\right)^2 = \frac{1}{2}\left(\pi\left(\frac{R}{2}\right)^2\sigma_1\right)\left(\frac{R}{2}\right)^2 = \frac{1\pi}{32}\sigma_1 R^4$$ 1

• $$I_{S_1'/G3} = I_{S_1'/C_1 3} + m_{S_1'}\vec{C_2G}^2 = \frac{1}{32}\pi\sigma_1 R^4 + \left(\pi\left(\frac{R}{2}\right)^2\sigma_1\right)\left(\frac{R}{4}\right)^2 = \frac{3}{64}\pi\sigma_1 R^4$$ 1

finalement • $$I_{S_1/G3} = I_{S_0/G3} - I_{S_1'/G3} = \frac{9}{16}\pi\sigma_1 R^4 - \frac{3}{64}\pi\sigma_1 R^4 = \frac{33}{64}\pi\sigma_1 R^4$$ 1

2) $I_{S_2/G_3} = \frac{3}{64}\pi\sigma_2 R^4 = \frac{3}{64}\pi(5\sigma_1)R^4 = \frac{15}{64}\pi\sigma_1 R^4$ / 2

question I

3) $I_{S/G_3} = I_{S_1/G_3} + I_{S_2/G_3} = \left(\frac{33}{64}+\frac{15}{64}\right)\pi\sigma_1 R^4 = \frac{3}{4}\pi\sigma_1 R^4 = \frac{3}{8}MR^2$ 2

Has Barème

4) $E_k = \frac{1}{2}M\vec{v}_{G/R}^{\,2} + \frac{1}{2}I_{S/G_3}\Omega^2$ 1 avec $I_{S/G_3} = \frac{3}{8}MR^2$; $\Omega = \dot{\theta}$

et $\vec{v}_{G/R} = \vec{v}_{C_1/R} + \vec{\Omega}\wedge\vec{C_1G}$

$= v_0\vec{e_x} + (\Omega\vec{e_z})\wedge\left(\frac{R}{4}\sin\theta\vec{e_x} - \frac{R}{4}\cos\theta\vec{e_y}\right)$

$= v_0\vec{e_x} + \frac{R}{4}\Omega\left(\sin\theta\vec{e_y} + \cos\theta\vec{e_x}\right)$

$= \left(-R\Omega + \frac{R\Omega}{4}\cos\theta\right)\vec{e_x} + \frac{R}{4}\Omega\sin\theta\vec{e_y}$

$= \frac{R\dot{\theta}}{4}(\cos\theta - 4)\vec{e_x} + \frac{R\dot{\theta}}{4}\sin\theta\vec{e_y}$

$\vec{v}_{G/R}^{\,2} = \left(\frac{R\dot{\theta}}{4}\right)^2\left((\cos\theta-4)^2+\sin^2\theta\right) = \left(\frac{R\dot{\theta}}{4}\right)^2(17-8\cos\theta)$ 1

$\Rightarrow E_k = \frac{1}{2}M\frac{R^2\dot{\theta}^2}{16}(17-8\cos\theta) + \frac{1}{2}\left(\frac{3}{8}MR^2\right)\dot{\theta}^2 = \frac{1}{2}\left(\frac{MR^2}{16}\right)(23-8\cos\theta)\dot{\theta}^2$ 1